Turning Weyl's tile argument into a no-go theorem

based on
Velocity polytopes of periodic graphs, arXiv:1109.1963

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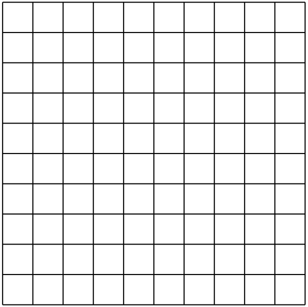
Experimental Search for Quantum Gravity: the hard facts
Perimeter Institute, Oct 2012

Overview

- Proposals for discrete spacetime have problems with Lorentz invariance, but also with isotropy of space.
- Maybe this is only because we haven't found the right models yet?
- I will present a mathematically rigorous no-go theorem which shows that the answer is *No* in a certain sense.
- So far, the theorem only applies to classical point particles.
- Advantages of mathematical rigor:
  - Requires making explicit definitions
  - Clear-cut assumptions
  - You can't argue against it!

Weyl's tile argument

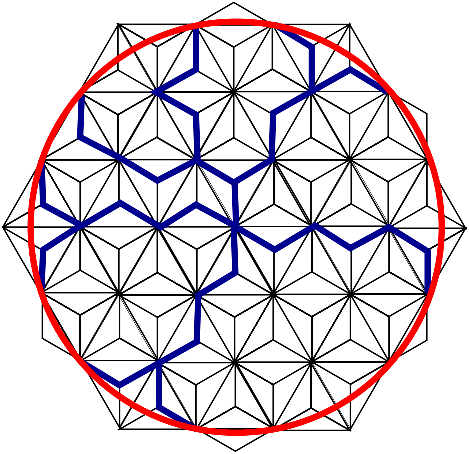


"If a square is built up of miniature tiles, then there are as many tiles along the diagonal as there are along the side; thus the diagonal should be equal in length to the side."

—Hermann Weyl,
Philosophy of Mathematics and Natural Science, 1927/49

Beyond square lattices

Maybe this is only because a square lattice is too simplistic?



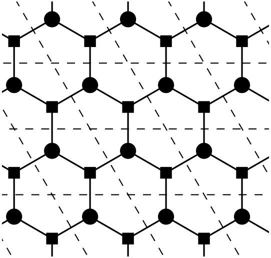
—Giacomo Mauro D'Ariano, *A Quantum-Digital Universe*, 2011

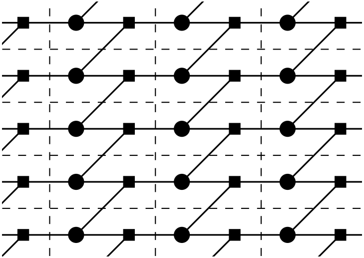
Formalizing the question I: Periodic graphs

Definition

A **periodic graph** is a graph G together with a free action of the discrete translation group $\mathbb{Z}^D$ with finitely many orbits.

$\longrightarrow$ No ambient space is needed!





Different embeddings of the same periodic graph!

Formalizing the question II: Trajectories

Question

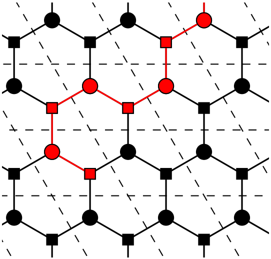
Can physics on a periodic graph be isotropic?

$\longrightarrow$ This may depend on the kind of physics considered.

In this talk: physics = classical point particles.

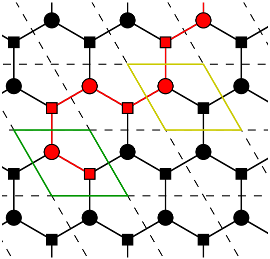
Definition

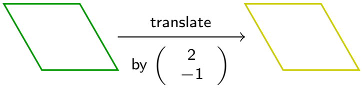
A **trajectory** is a sequence $x_0, x_1, x_2, \ldots$ with $x_t \in G$, indexed by discrete time $t \in \mathbb{N}$, such that every x_t is adjacent to x_{t+1}.



Formalizing the question III: Displacements

Let $C \subseteq G$ be a unit cell. Consider a trajectory $(x_t)_{t\in\mathbb{N}}$ with $x_0 \in C$.





Definition

The **displacement** is the translation $d_t \in \mathbb{Z}^D$ such that $x_t - d_t \in C$.

Formalizing the question IV: Velocities

Definition

The trajectory's **(macroscopic) velocity** $v \in \mathbb{R}^D$ is

$$v = \lim_{t \to \infty} \frac{d_t}{t},$$

if the limit exists.

Proposition

Velocities do not depend on the choice of unit cell.

$\longrightarrow$ No notion of distance is required in order to talk about velocities!

Main result

The set of velocities is a subset of $\mathbb{R}^D$.

$\longrightarrow$ Isotropy would mean that this set is the unit ball of some Euclidean metric, i.e. an ellipsoid.

Theorem

If G is connected, then the set of velocities is a **convex polytope** in $\mathbb{R}^D$.
Furthermore, there is a simple algorithm for computing the vertices of that polytope.

$\longrightarrow$ The continuum limit of a periodic graph, as experienced by a classical point particle, **cannot be isotropic**.