Disentangling influence and inference in quantum and classical theories

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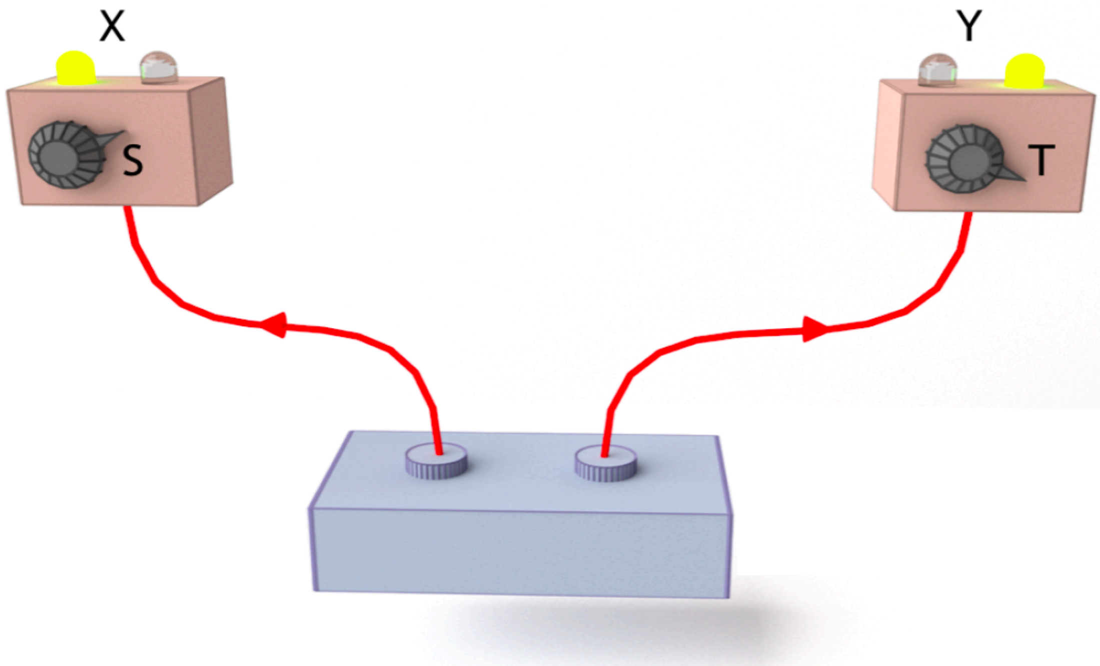
The quantum omelette of ontological and epistemological concepts

"[...] our present QM formalism is not purely epistemological; it is a peculiar mixture describing in part realities of Nature, in part incomplete human information about Nature all scrambled up by Heisenberg and Bohr into an omelette that nobody has seen how to unscramble. Yet we think that the unscrambling is a prerequisite for any further advance in basic physical theory. For, if we cannot separate the subjective and objective aspects of the formalism, we cannot know what we are talking about; it is just that simple"

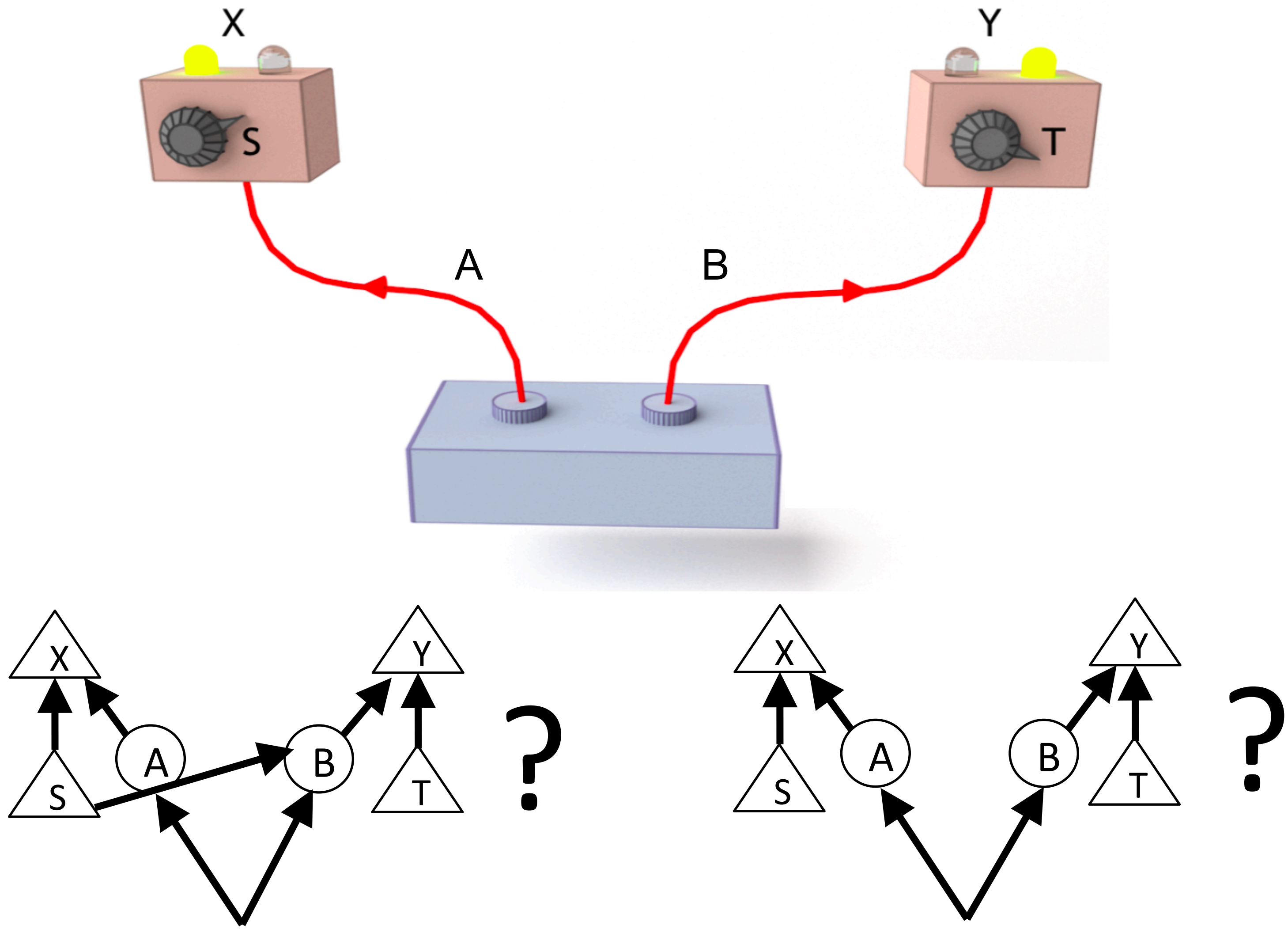
— E.T. Jaynes, 1989

"realities of nature" = **causal relations**
"incomplete human information about nature" = **inferential relations**



P(X,Y|S,T)

	X=0, Y=0	X=0, Y=1	X=1, Y=0	X=1, Y=1
S=0, T=0	0.427	0.073	0.073	0.427
S=0, T=1	0.427	0.073	0.073	0.427
S=1, T=0	0.427	0.073	0.073	0.427
S=1, T=1	0.073	0.427	0.427	0.073


X
Y
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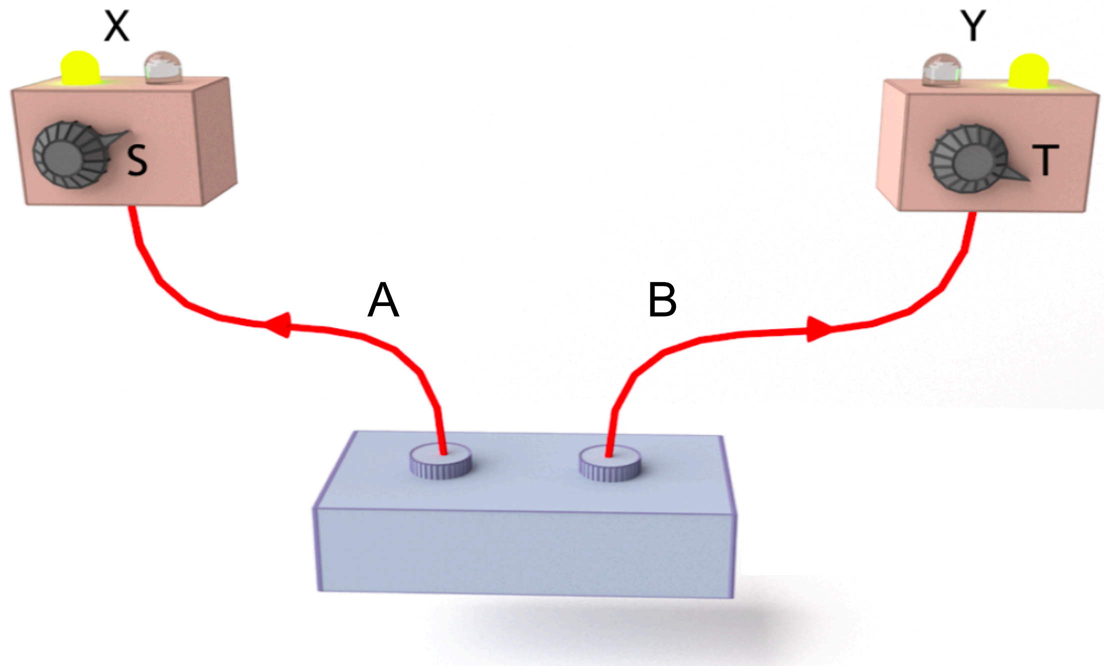
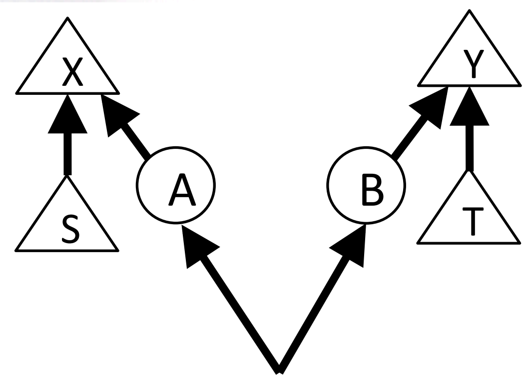
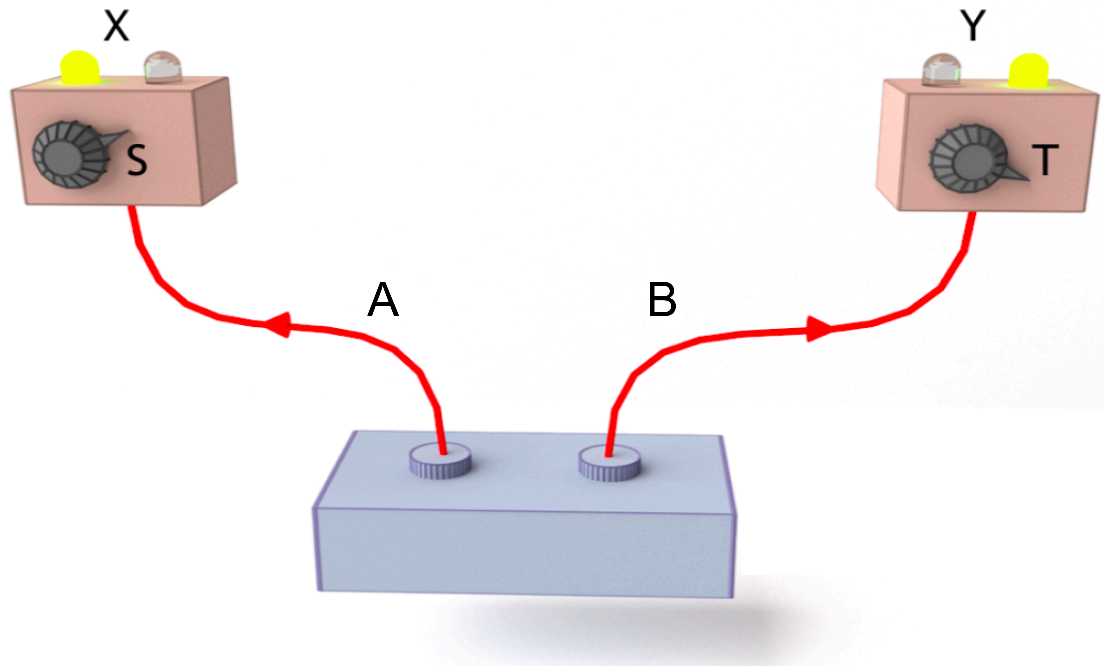
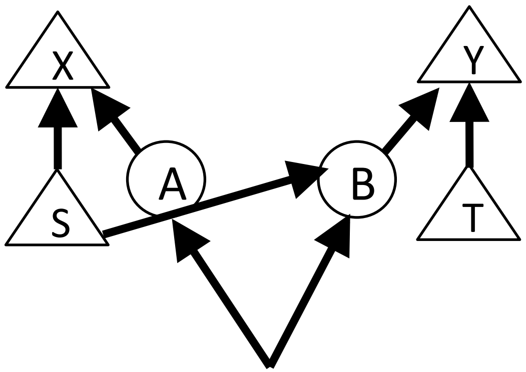


The conservative causal hypothesis



But the statistical correlations predicted by quantum theory *violate* Bell inequalities (which follow from assuming this causal hypothesis and a classical theory of inference)



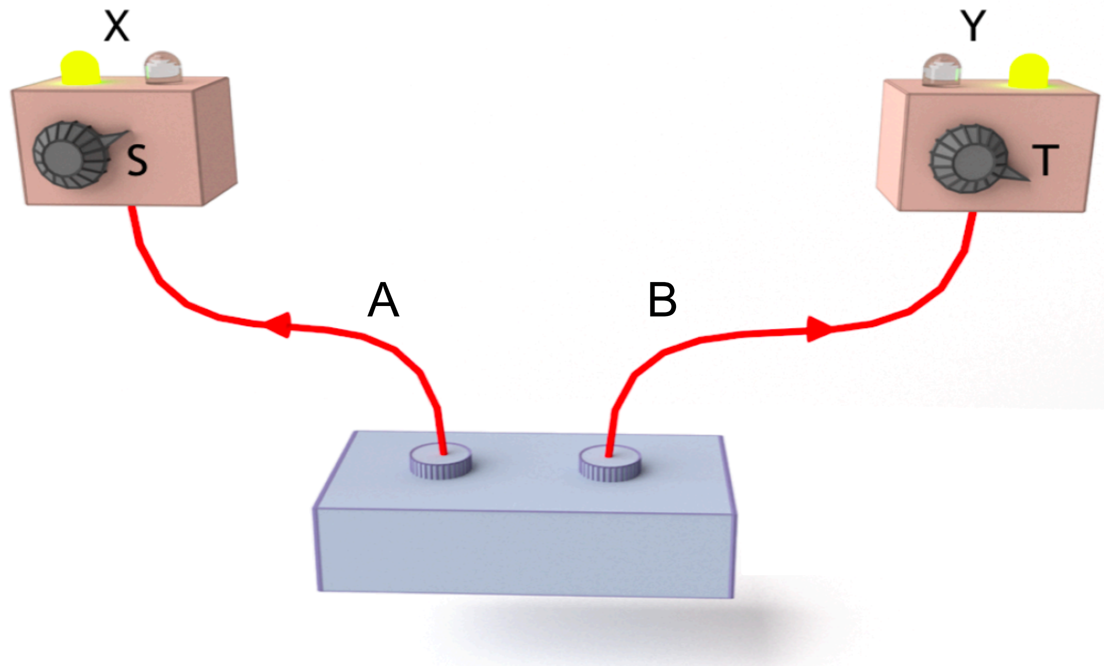


The radical causal hypothesis

But: Relativity theory → no causal influence between the wings

Also: No fine-tuning → no causal influence between the wings

Wood and RWS, New J. Phys. 17, 033002 (2015)



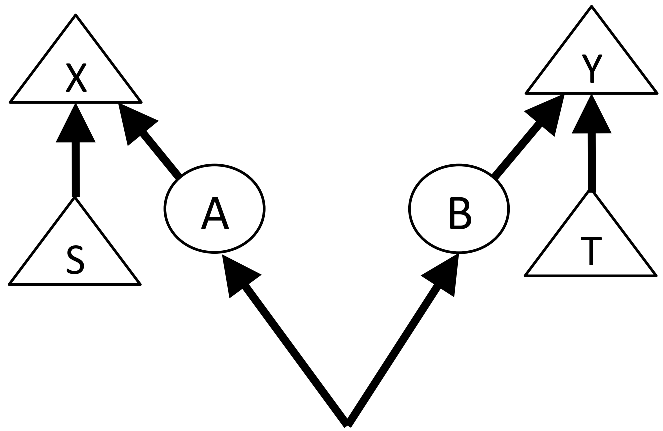
We still need to provide a causal explanation of the the experimental statistics

The research program which I favour:
Quantum Theory is **causally conservative** but **inferentially radical**

Given:

$P(AB)$

$P(X|AS)$



Given:

ρ_{AB}

$\rho_{X|SA}$

Bayesian updating

$P(B) \to P(B|SX)$

Bayesian inversion

$$P(A|SX) = \frac{P(X|AS)P(A)}{P(X|S)}$$

Conditional from joint

$$P(B|A) = \frac{P(AB)}{P(A)}$$

Belief propagation

$$P(B|SX) = \sum_A P(B|A)P(A|SX)$$

Bayesian updating

$\rho_B \to \rho_{B|SX}$

Bayesian inversion

$$\rho_{A|XS} = \rho_{X|AS} \star \rho_A \rho_{X|S}^{-1}$$

Conditional from joint

$$\rho_{B|A} = \rho_{AB} \star \rho_A^{-1}$$

Belief propagation

$$\rho_{B|SX} = \mathrm{tr}_A(\rho_{B|A}\rho_{A|SX})$$

Leifer & RWS, PRA 88, 052130 (2013)

But there are many problems with this approach

See:

Leifer & RWS, PRA 88, 052130 (2013)

Horsman, Heunen, Pusey, Barrett, RWS, Proc. R. Soc. A 473 20170395 (2017)

To propose a quantum generalization of inference, it helps to have a **synthetic approach to theories of inference**

Coecke & RWS, Synthese 186, 651 (2012)
Cho & Jacobs. Math. Structures Comput. Sci. 29. 938 (2019)
Fritz, Advances in Mathematics 370, 107239 (2020)

But there is some preparatory unscrambling that needs to be done first

Motivations for our formalism that will **not** be discussed here:

Disentangling causal and inferential notions in:
- Operational theories
- Ontological models of operational theories

A categorical formalization of a notion of classicality for ontological models termed “generalized noncontextuality”

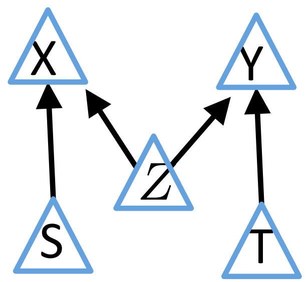
Motivations from the field of causal inference

The standard framework used in this field
also scrambles influence and inference somewhat

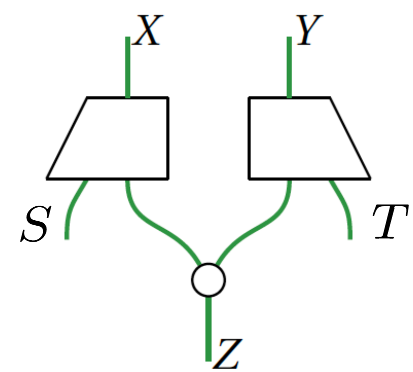
(We’ll return to this near the end)

Some assumptions:

Directed Acyclic Graph (DAG)



String diagram

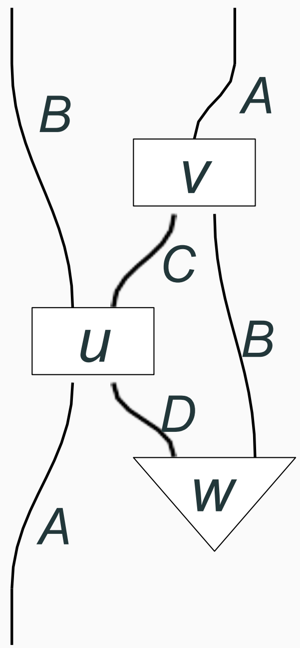


Probabilities are always epistemic

For the rest of the talk:
All systems are classical
All variables are discrete

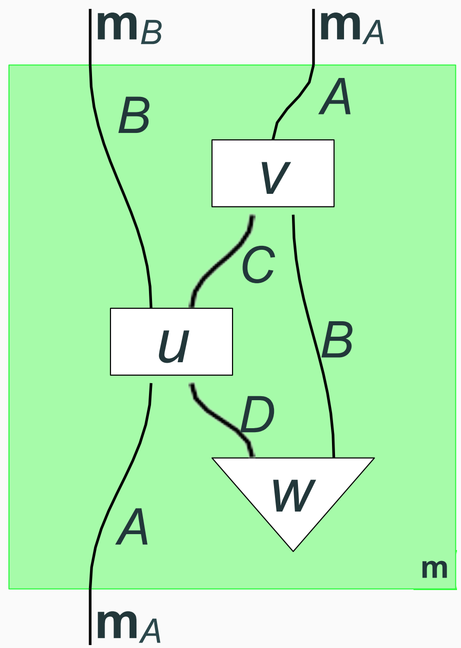
Aim: to disentangle causal relations and inferential relations

Tools: Process theories



Aim: to disentangle causal relations and inferential relations

Tools: Process theories and Diagram-Preserving maps



Aim: to disentangle causal relations and inferential relations

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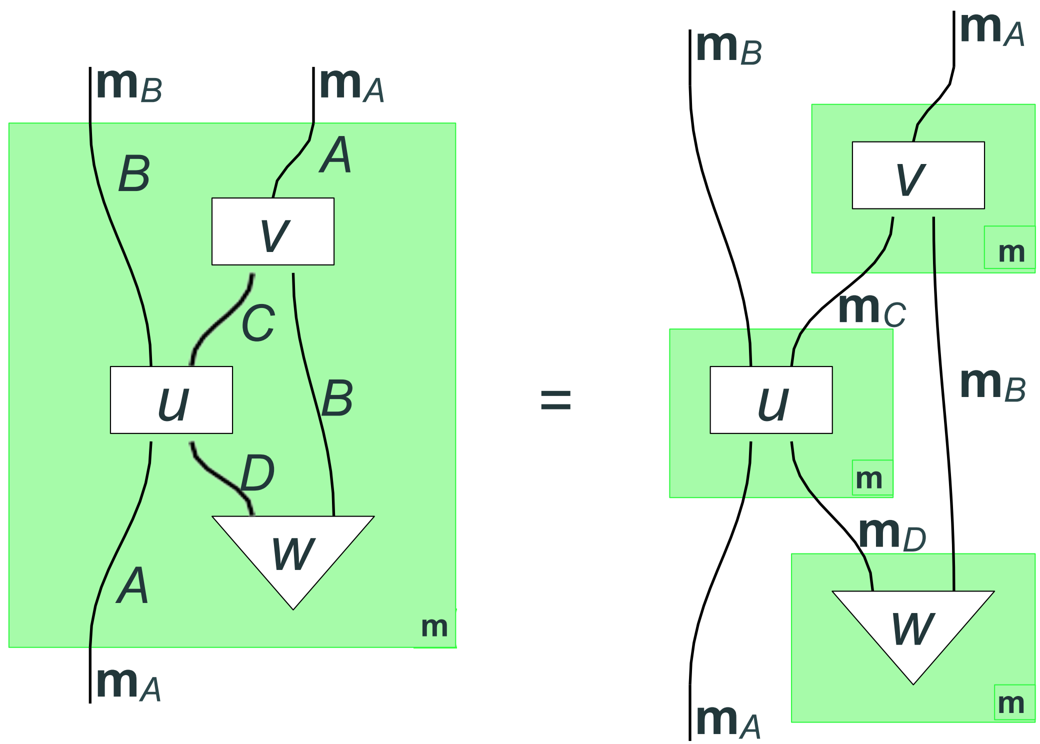


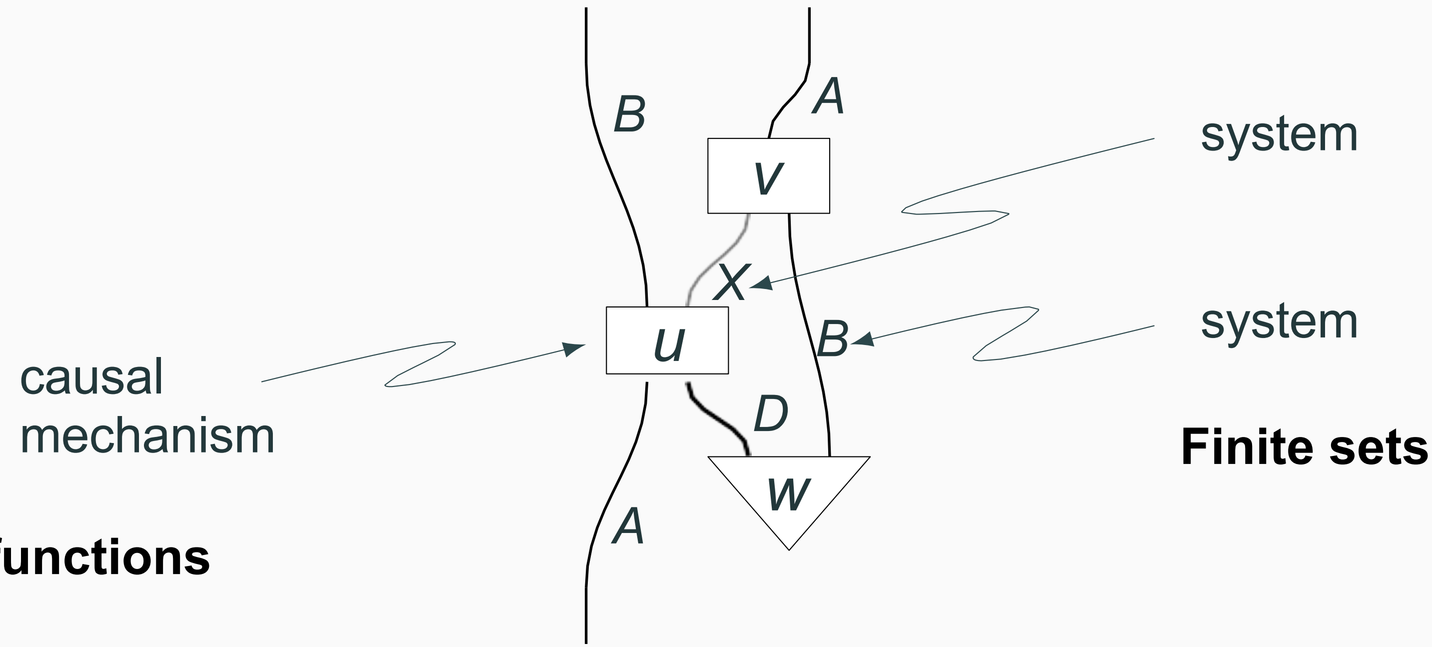
Causal-Inferential Framework

Causal: “realities of Nature”

Inferential: “incomplete human information about Nature”

Causal process theory, CAUS

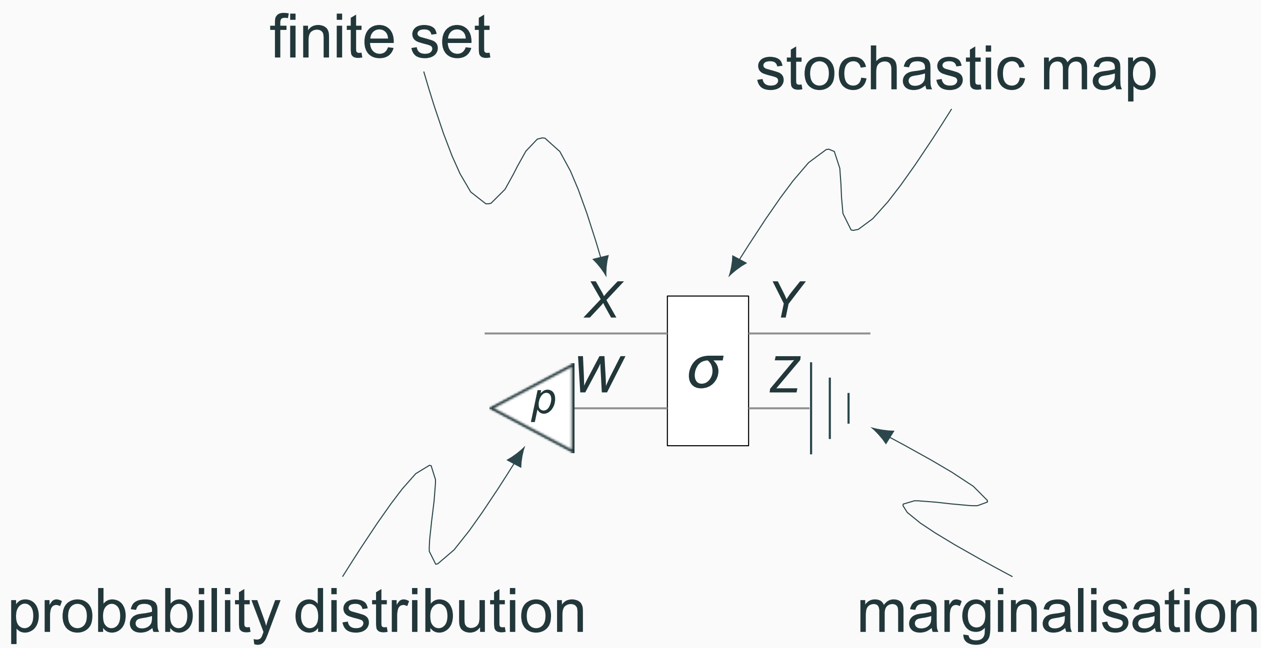
– hypothesis about the fundamental systems (the causal mediaries) and the causal mechanisms relating them



(The SMC FINSET)

Inferential process theory, INF

i) Bayesian probability theory, BAYES

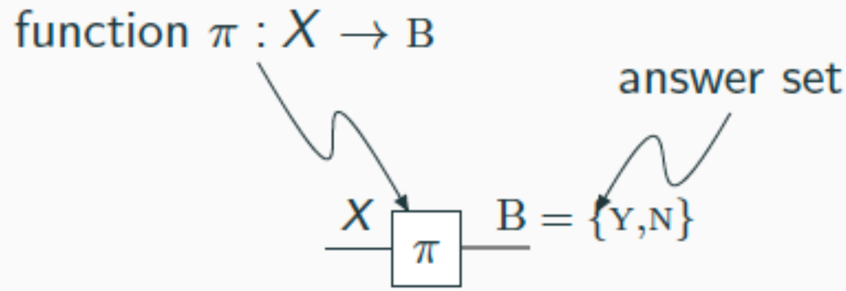


(The SMC FINSTOCH)

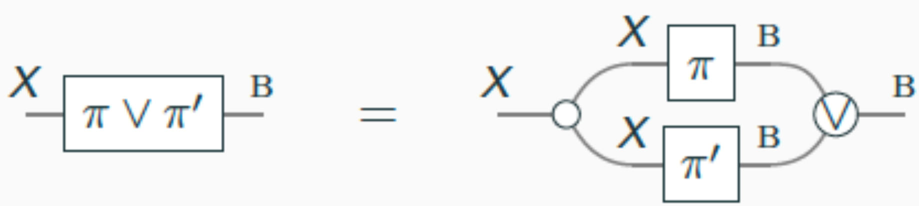
Inferential process theory, INF

ii) Boolean propositional logic, BOOLE

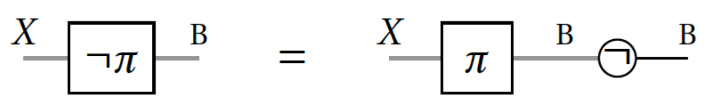
- Propositions as yes/no questions:



- Logical operations:

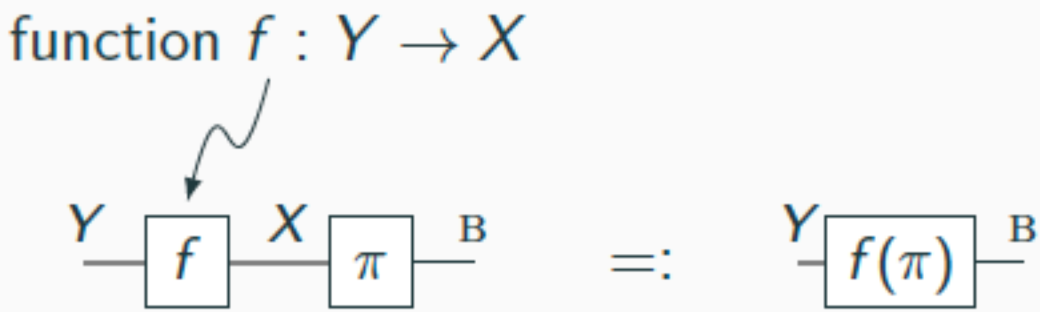


where $\vee(\mathrm{Y},\mathrm{Y}) = \vee(\mathrm{Y},\mathrm{N}) = \vee(\mathrm{N},\mathrm{Y}) = \mathrm{Y}$, $\vee(\mathrm{N},\mathrm{N}) = \mathrm{N}$.

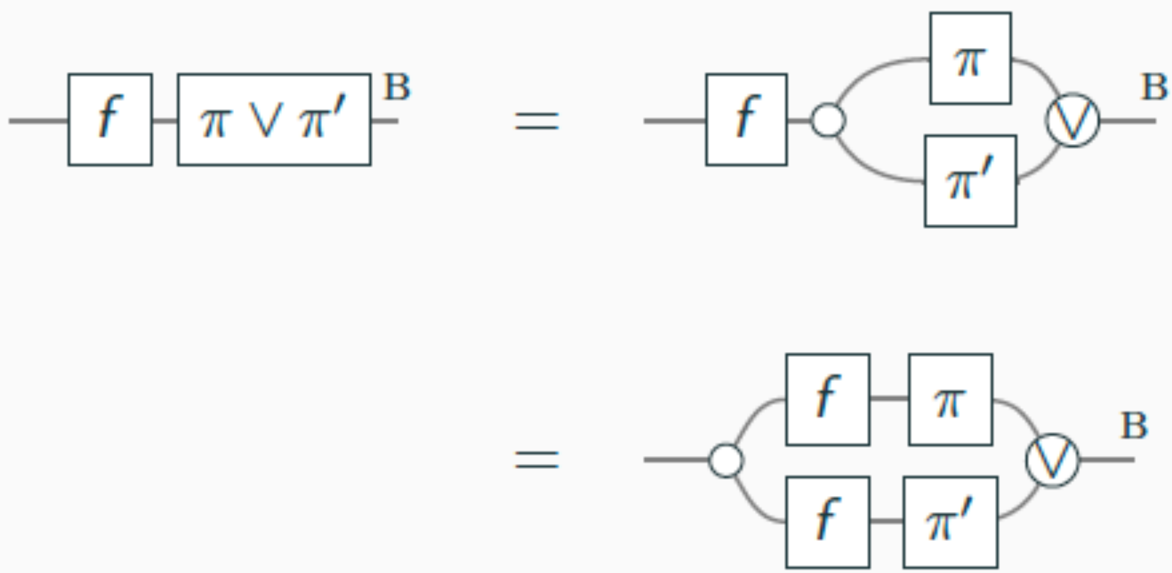


where $\neg\mathrm{Y} = \mathrm{N}$, $\neg\mathrm{N} = \mathrm{Y}$

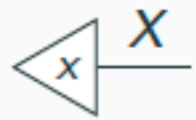
- Boolean algebra homomorphisms:



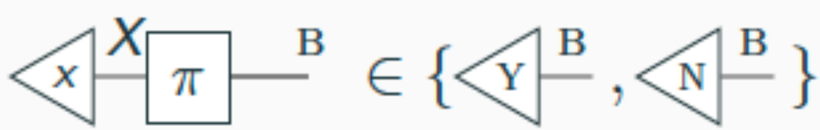
e.g. f preserves $\vee$:



- Value assignments:

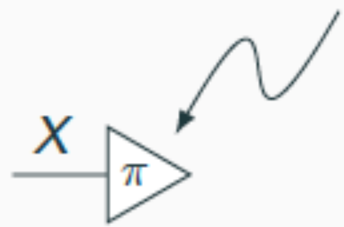


  - assigns value x to variable X
  - assigns answer Y, N to propositional questions:



- Propositions as effects:

partial function $\pi : X \to \star$

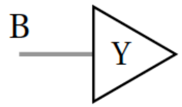


- Truth values as scalars:

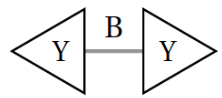
$$\{\textsc{True}, \textsc{False}\}$$

where $\textsc{True}(*) = *$ and $\textsc{False}(*) = $ *undefined*.

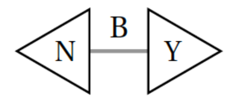
We can define Boolean “effects”



such that

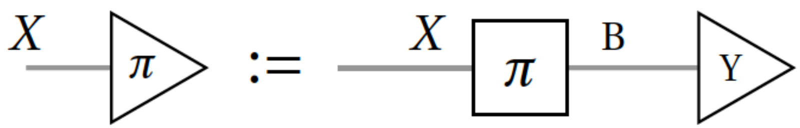


= TRUE and

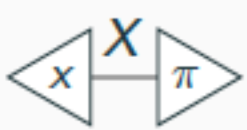


= FALSE

So we can define the effect associated with the proposition π by



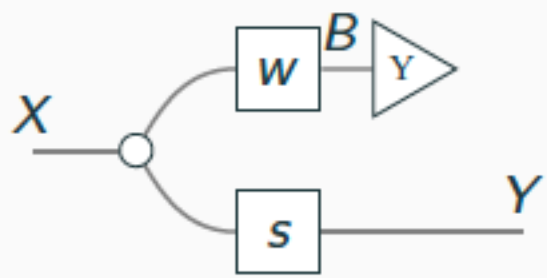
A value assignment of x to X provides a truth value assignment to a proposition about X



$\in \{\text{TRUE}, \text{FALSE}\}$

$$\textsc{Inf} = \textsc{Bayes} + \textsc{Boole}$$

- what processes can we create?
  i. Note that $\textsc{Bayes}, \textsc{Boole} \subseteq \textsc{SubStoch}$
  ii. Any substochastic map can be realised by:

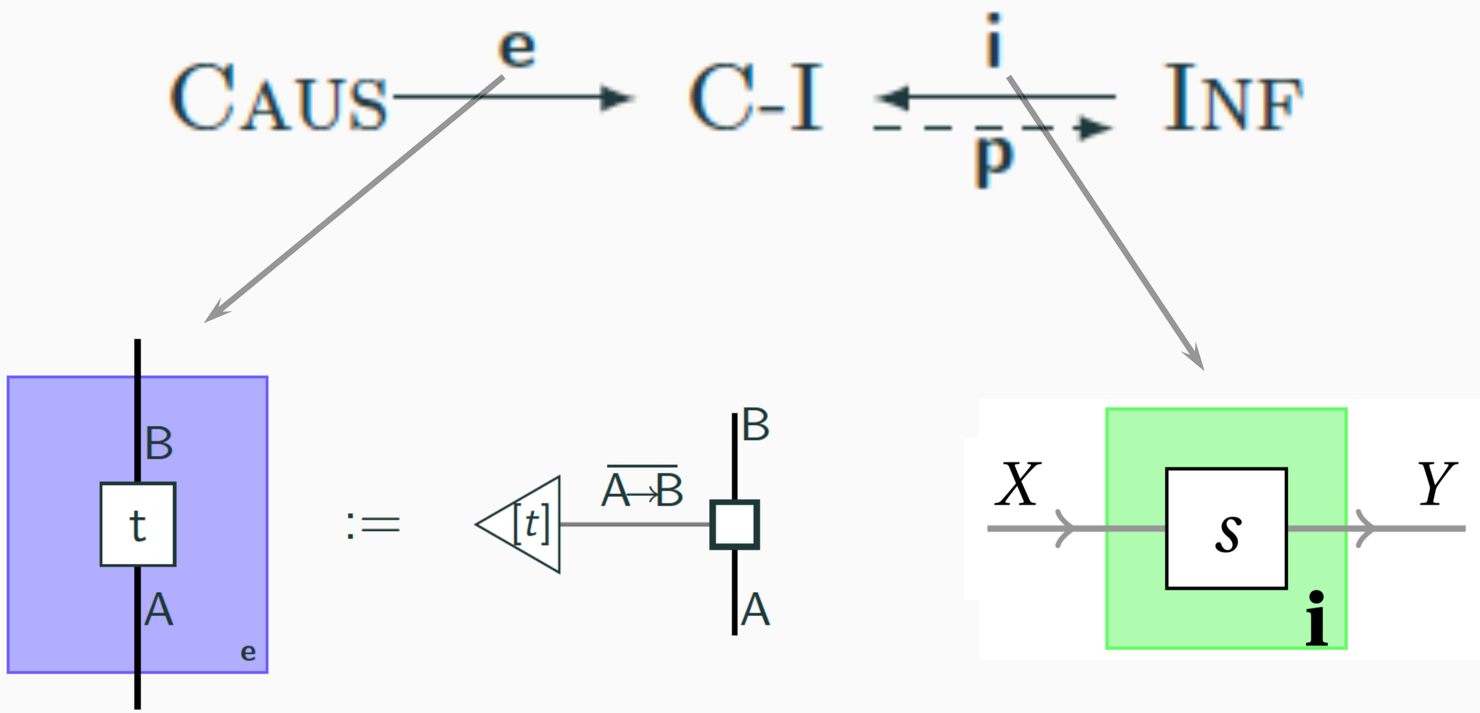


where $s, w \in \textsc{Bayes}$ and $\curlyvee \in \textsc{Boole}$

$$\implies \textsc{Inf} = \textsc{SubStoch}$$

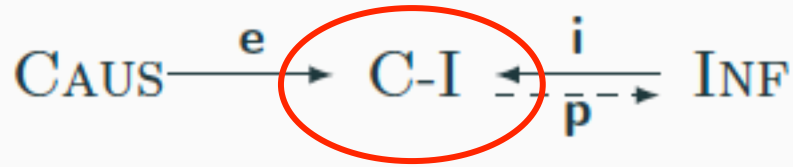
Causal-inferential process theory, C-I

- systems from CAUS and INF are systems in C-I



Notate point distribution as [t]

Causal inferential theories



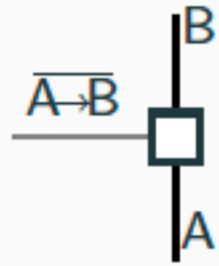
- three generators (and their rewrite rules) define the interaction between systems from CAUS and systems from INF.

Interaction 1 "what we know"

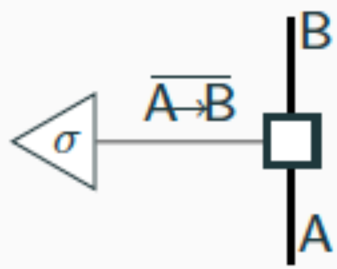
- Denote set of causal mechanisms transforming A into B by

$$\overline{A \rightarrow B}$$

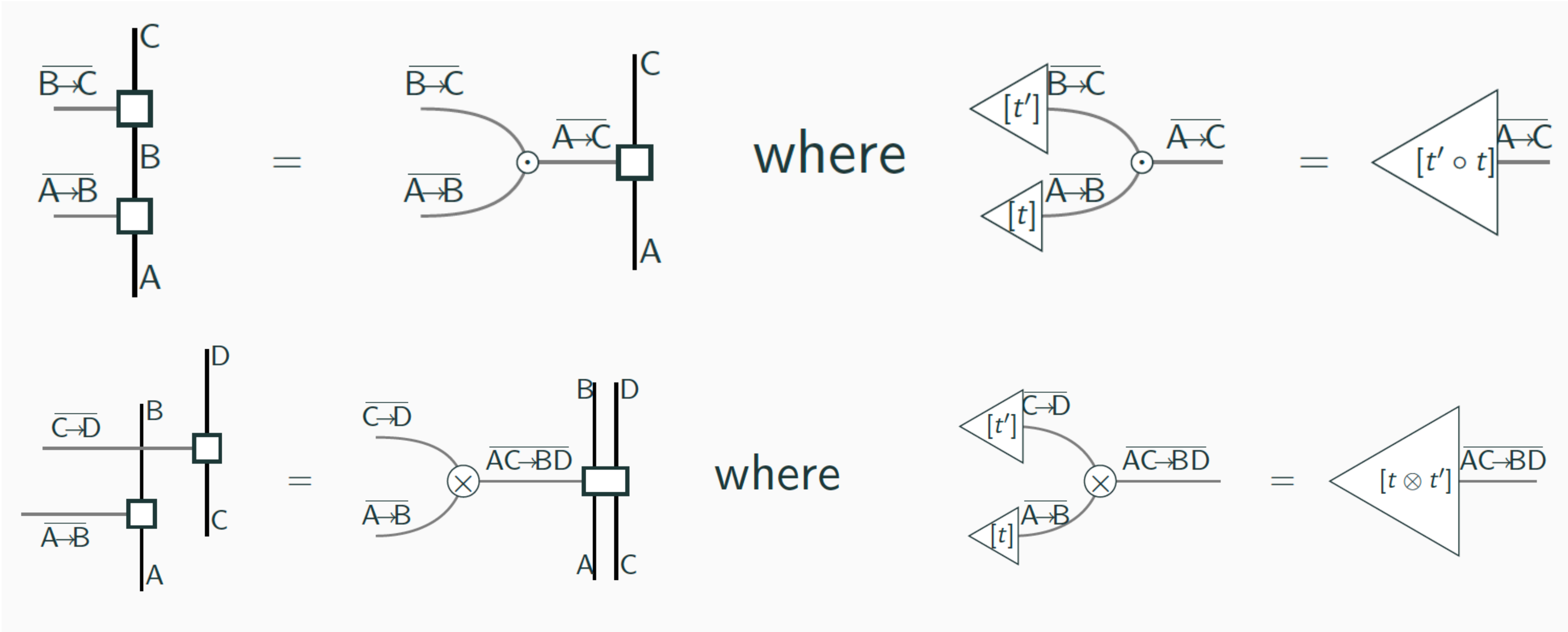
- for each pair of causal mediaries (A, B) we have an interaction:



this lets us specify what we know about the causal mechanism transforming A into B,e.g.:

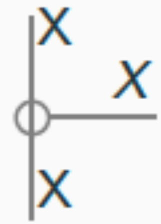


Interaction 1 – consistency conditions



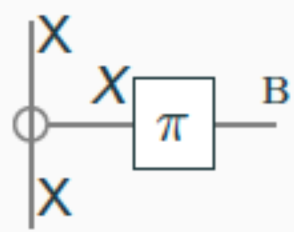
Interaction 2 "what we learn"

- For each classical system X we have an interaction:

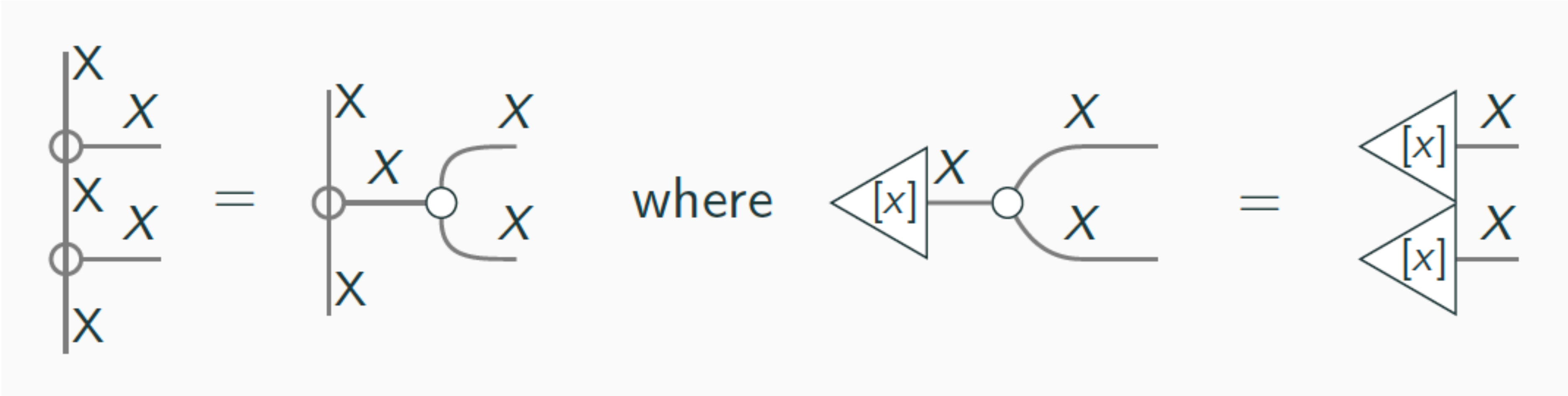


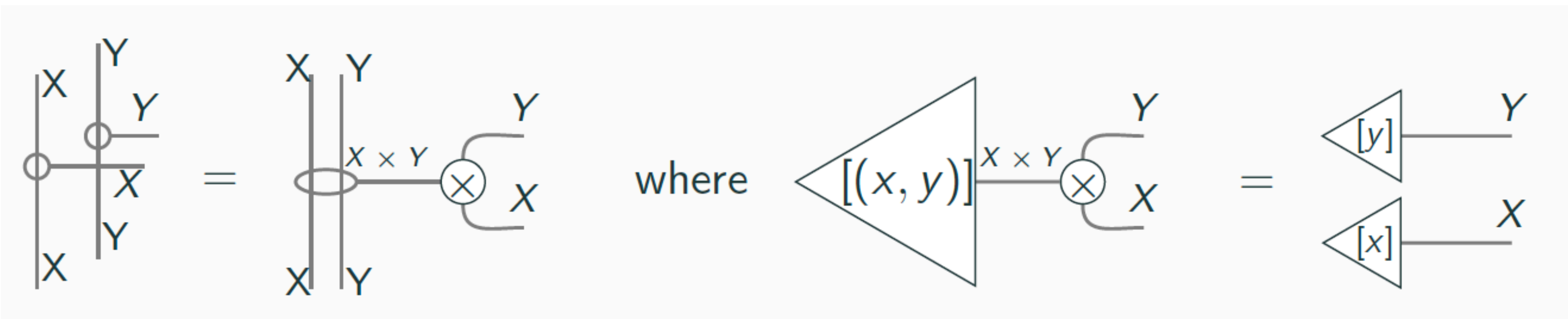
which represents learning about the state of the classical system X.

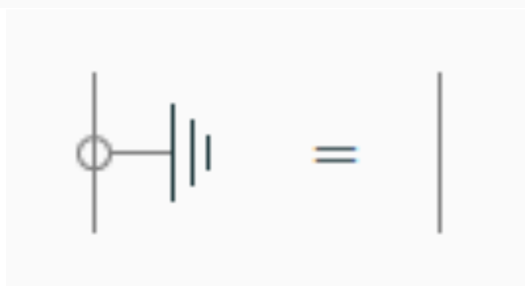
- For example, we can ask propositional questions about a classical system:



Interaction 2 – consistency conditions







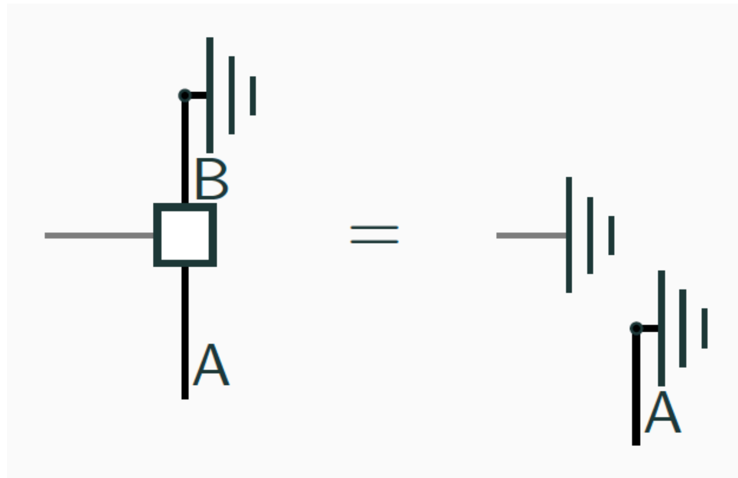
Interaction 3 "what we ignore"

- For each causal system we introduce an interaction

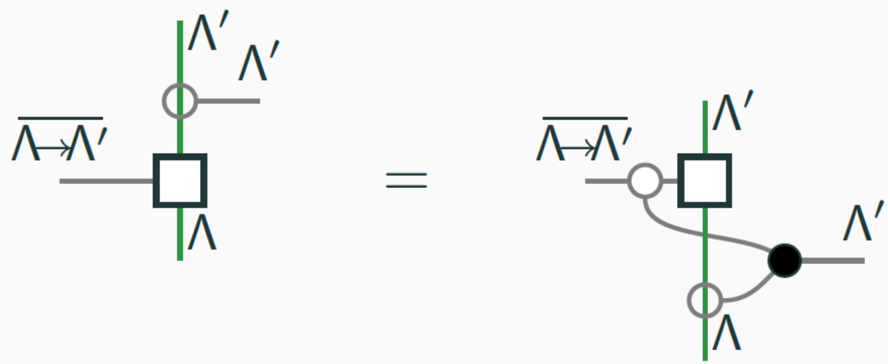
which represents ignoring a causal system.

- which satisfies the constraint:

Interaction 1 & 3 – consistency condition



- Extra rewrite rule:



where

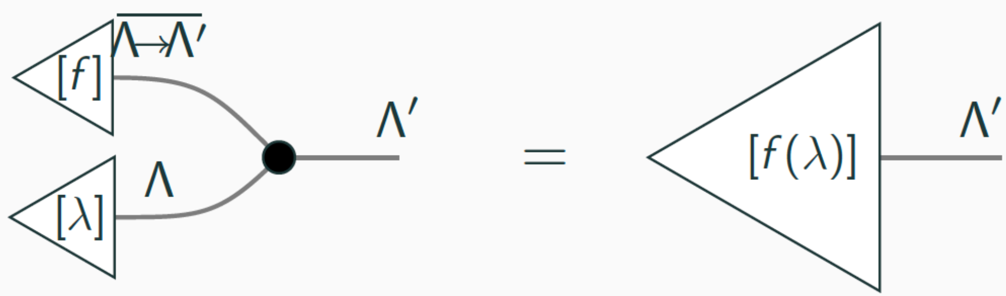
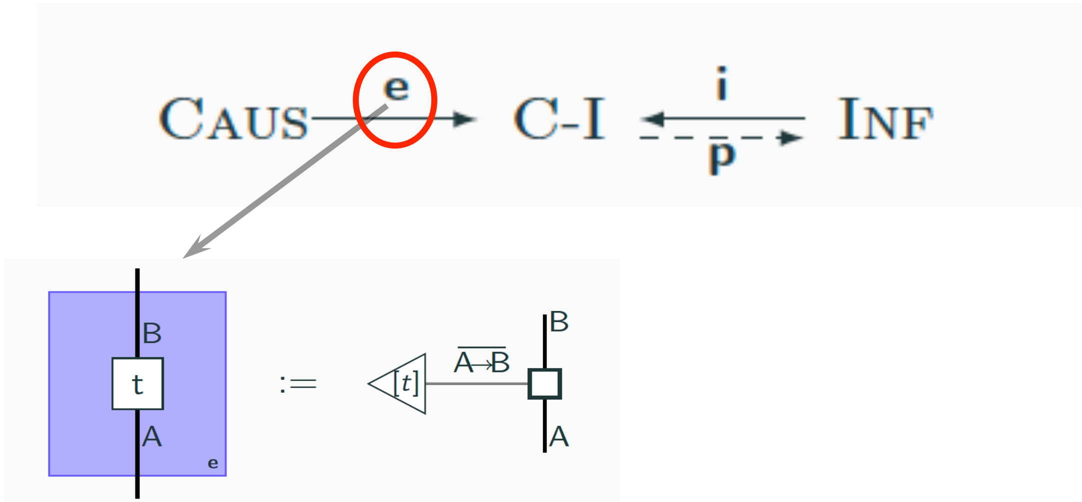
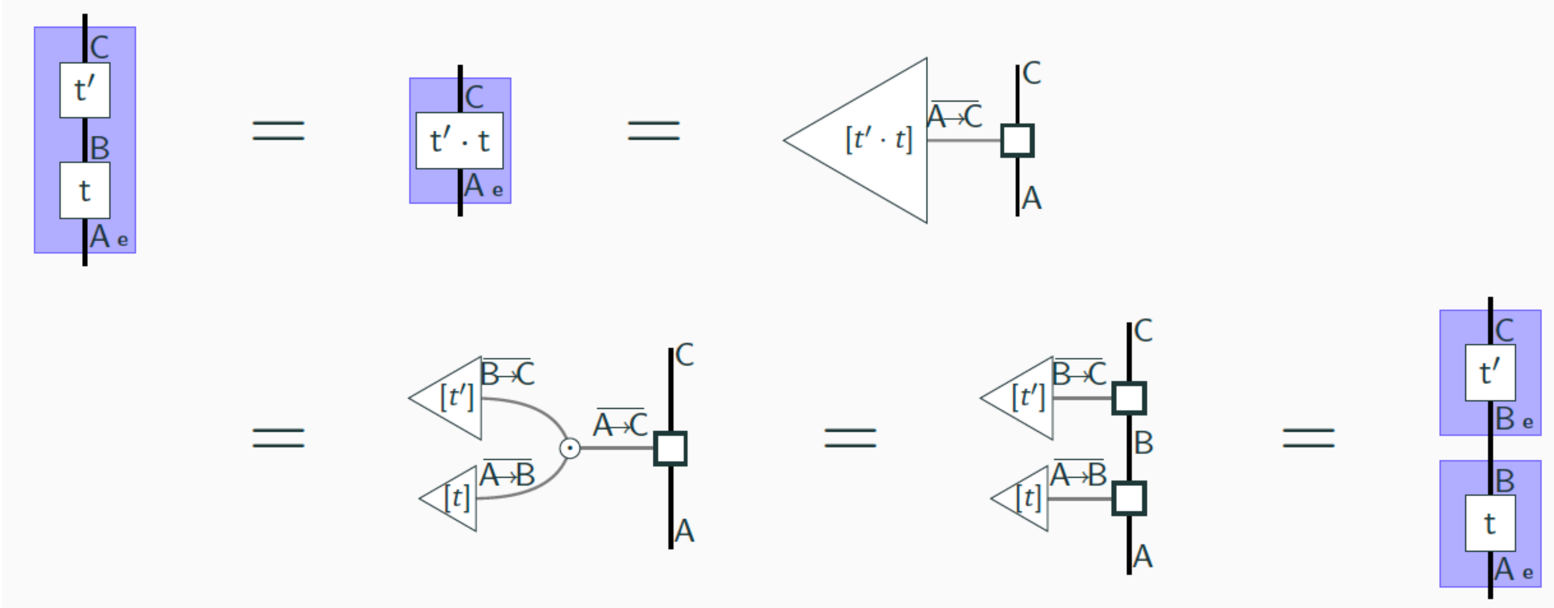


Diagram preserving map e : Caus → C-I



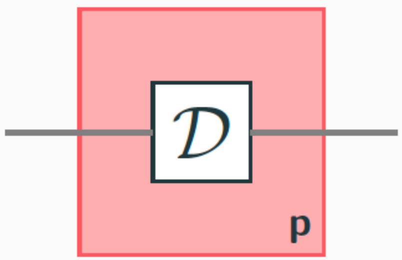
- To check this is diagram preserving (for example):



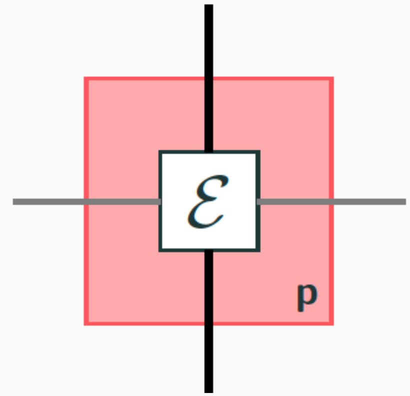
Predictions

$$\textsc{Caus} \xrightarrow{\;\mathbf{e}\;} \text{C-I} \;\underset{\mathbf{p}}{\overset{\mathbf{i}}{\rightleftarrows}}\; \textsc{Inf}$$

– diagram preserving partial map

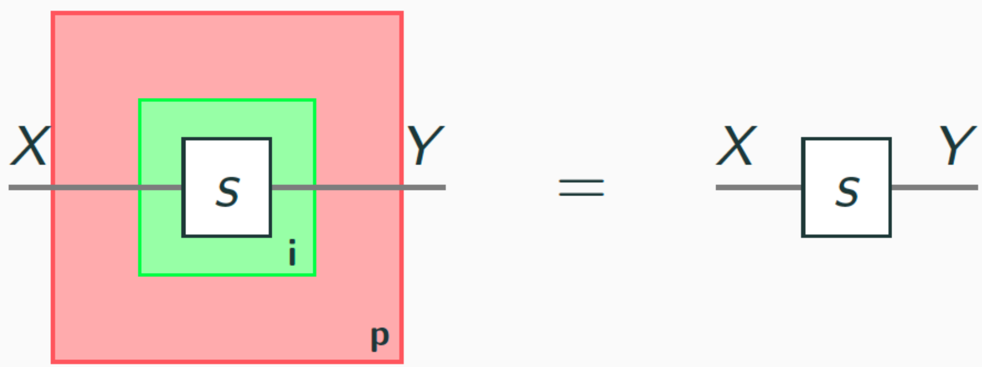


$\in$ $\textsc{Inf}$

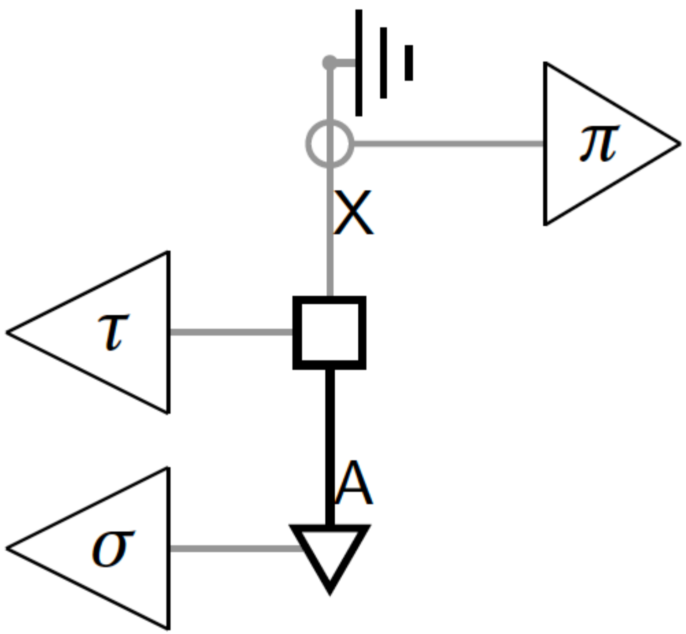


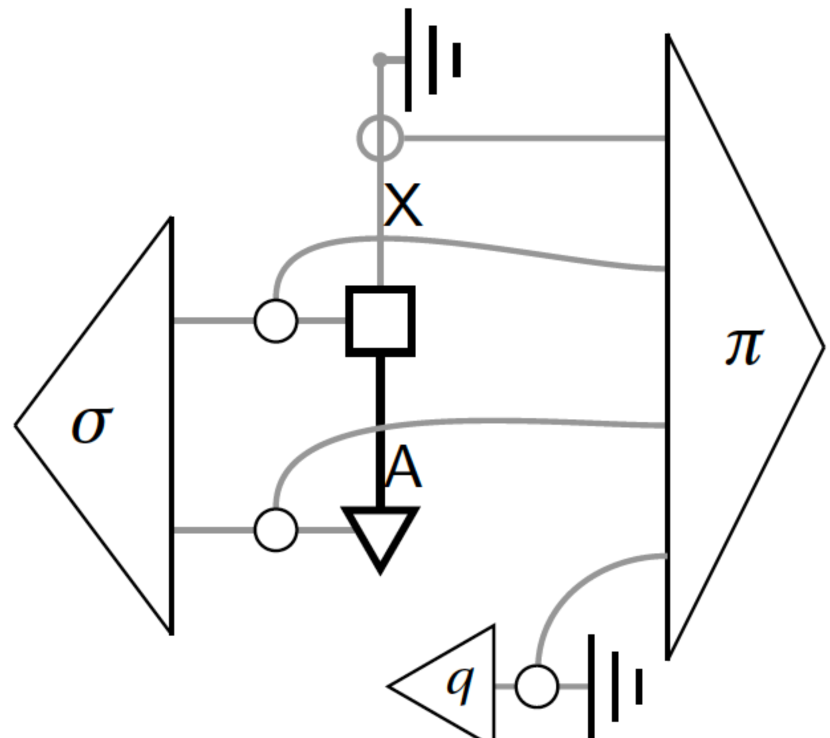
undefined

- Consistency condition:



Some examples of C-I diagrams

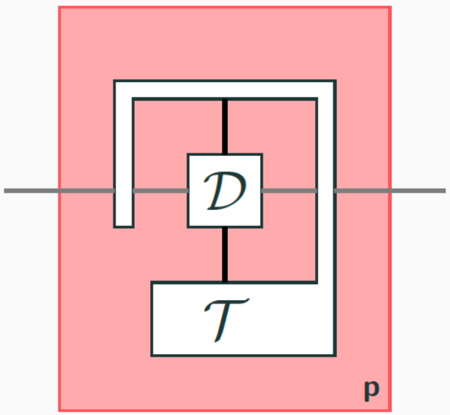




Inferential equivalence

– Two processes in C-I are inferentially equivalent iff they look the same from the perspective of INF.

How do we get from $\mathcal{D}$ to INF?

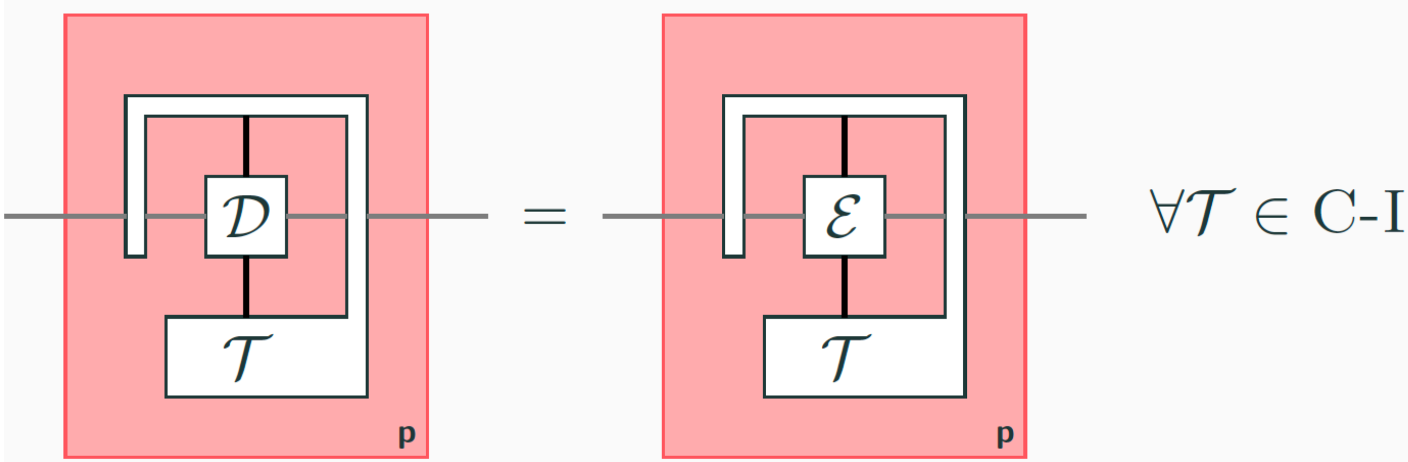


Inferential equivalence

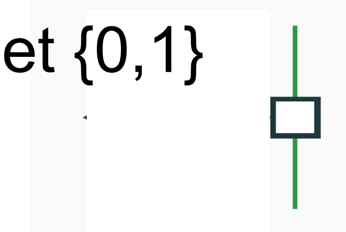
– Two processes in C-I are inferentially equivalent iff they look the same from the perspective of INF.

$$\mathcal{D} \sim_{\mathbf{p}} \mathcal{E}$$

if and only if



Consider the four functions on the set {0,1}

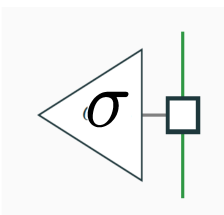


$$f_{\text{identity}}, f_{\text{flip}}, f_{\text{reset}-0}, f_{\text{reset}-1}$$

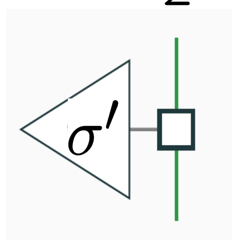
Now, consider the states of knowledge:

$$\sigma = \frac{1}{2}[f_{\text{identity}}] + \frac{1}{2}[f_{\text{flip}}]$$

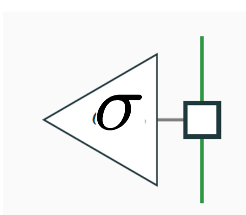
$$\sigma' = \frac{1}{2}[f_{\text{reset}-0}] + \frac{1}{2}[f_{\text{reset}-1}]$$



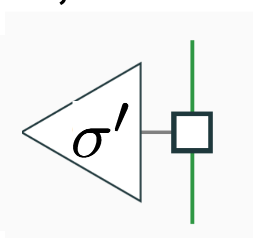
$\neq$

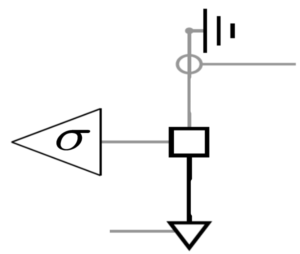


And yet,



$\sim_{\mathbf{p}}$





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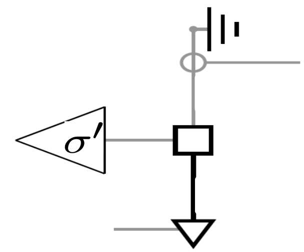


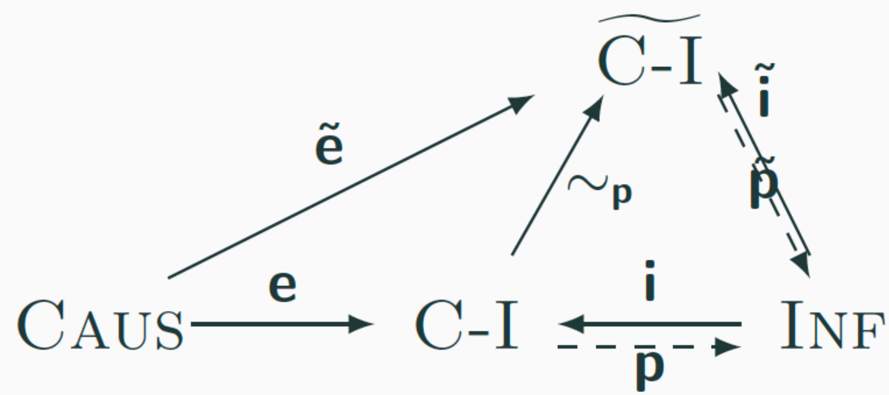
Both are associated to the stochastic matrix

$$\begin{pmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix} = \frac{1}{2}\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} + \frac{1}{2}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$= \frac{1}{2}\begin{pmatrix} 1 & 1 \\ 0 & 0 \end{pmatrix} + \frac{1}{2}\begin{pmatrix} 0 & 0 \\ 1 & 1 \end{pmatrix}$$

Quotienting

– This equivalence relation, $\sim_{\mathbf{p}}$, is preserved by composition, hence, we can quotient by it:



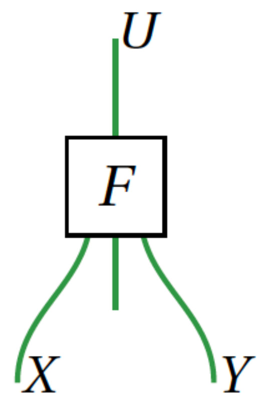
Applications to Causal Inference

The standard framework used in the field is not optimal for discriminating **claims about causal relations** and **claims about inferential relations**

Example of how our framework can help:

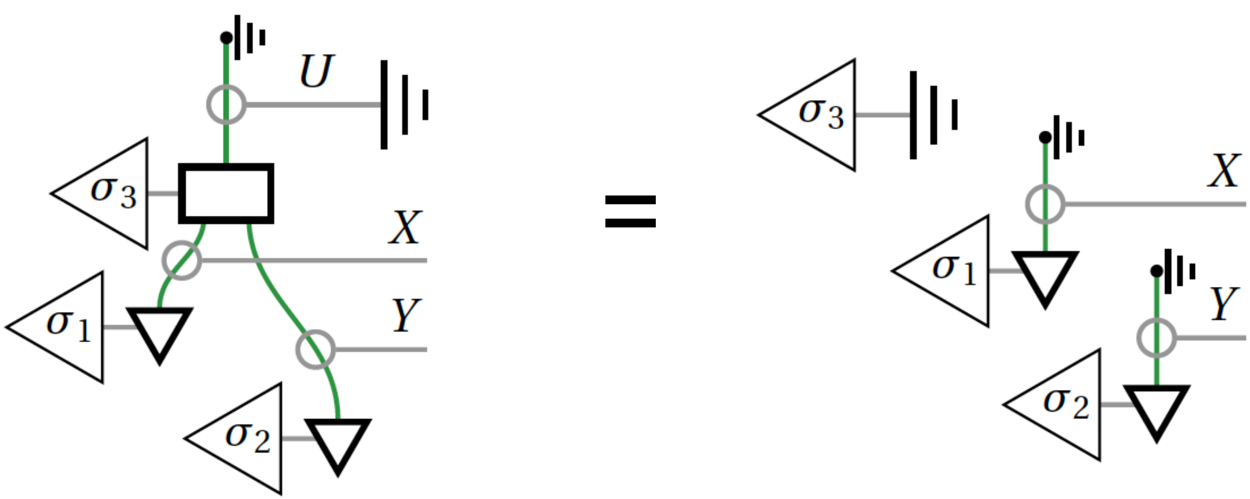
- Provide a graphical means of proving the “d-separation theorem” and generalizations thereof

If U is a common effect of X and Y (a collider)



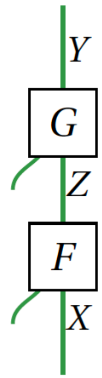
in CAUS

Then X and Y are independent given marginalization over U



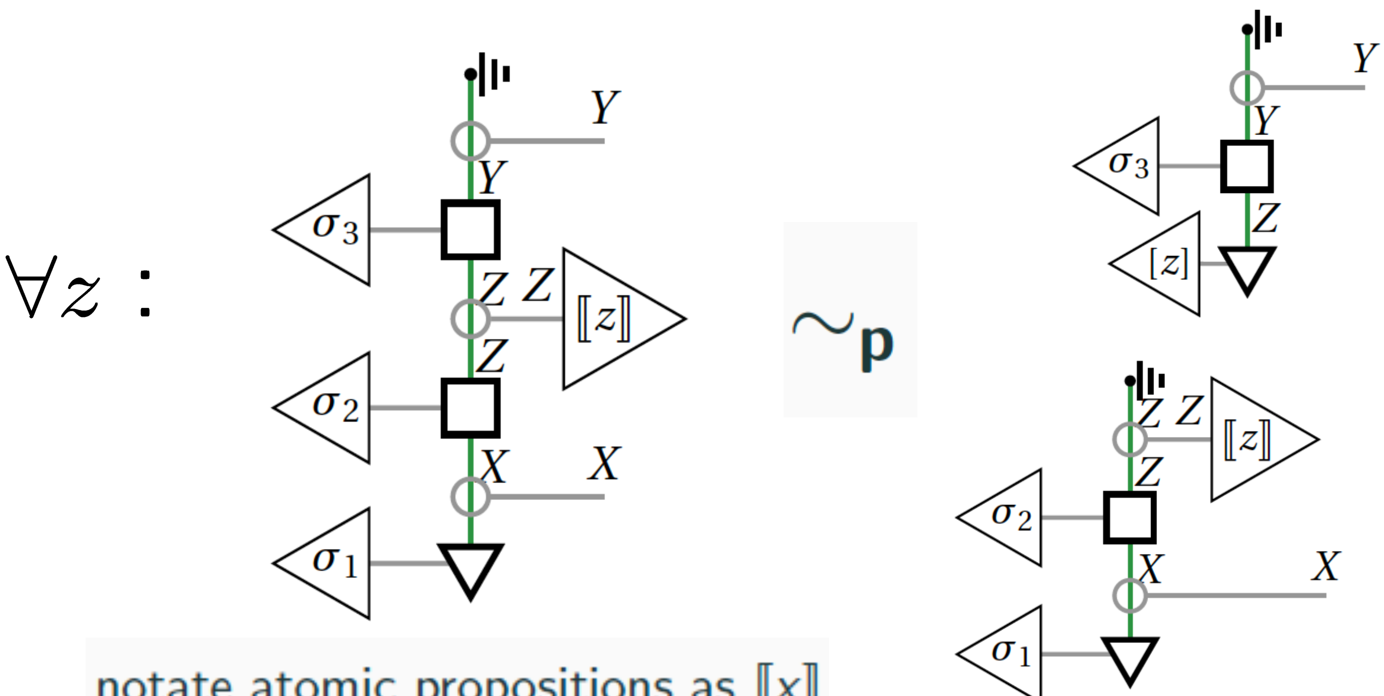
in C-I

If Z is the causal mediary between X and Y (chain)



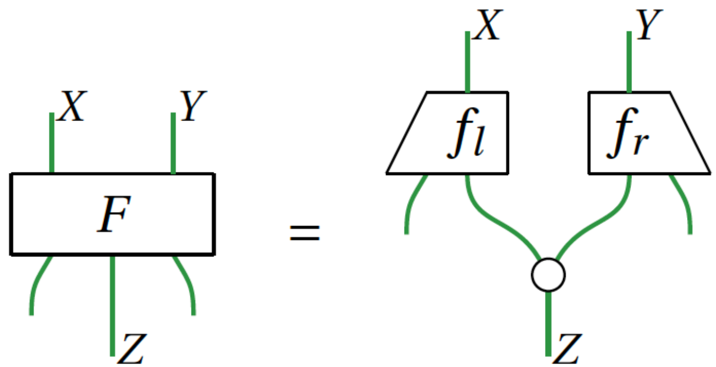
in CAUS

Then X and Y are conditionally independent given Z



notate atomic propositions as [[x]].

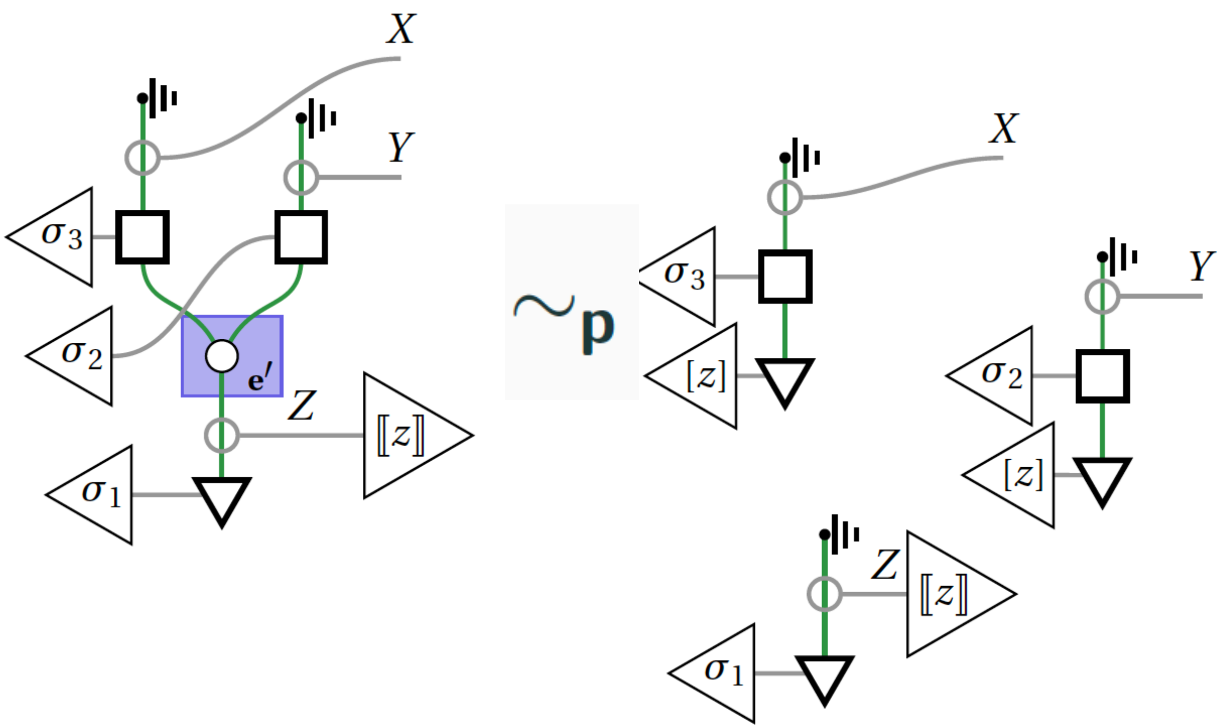
If Z is a common cause of X and Y (a fork)



in CAUS

Then X and Y are conditionally independent given Z

$\forall z$:



in C-I

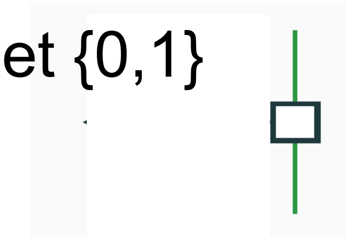
Generalized notions of conditional independence

Notion of independence of X and Y for **a given value** of Z

Notion of independence of X and Y for **all** states of knowledge of another variable Z

Any of these notions of independence of X and Y **relativized to a particular set of parameter values in the causal model** (functional dependences and states of knowledge)

Consider the four functions on the set {0,1}



$$f_{\text{identity}}, f_{\text{flip}}, f_{\text{reset}-0}, f_{\text{reset}-1}$$

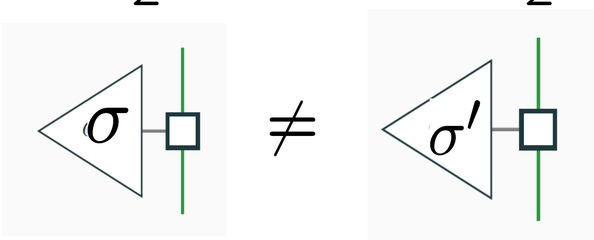
Now, consider the states of knowledge:

$$\sigma = \frac{1}{2}[f_{\text{identity}}] + \frac{1}{2}[f_{\text{flip}}]$$

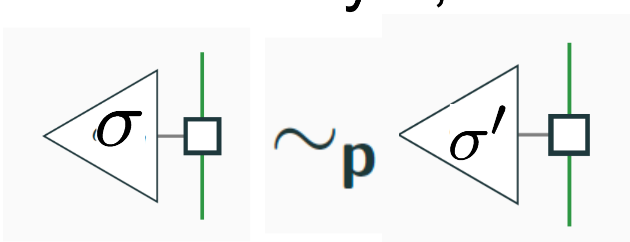
Perfect causal influence

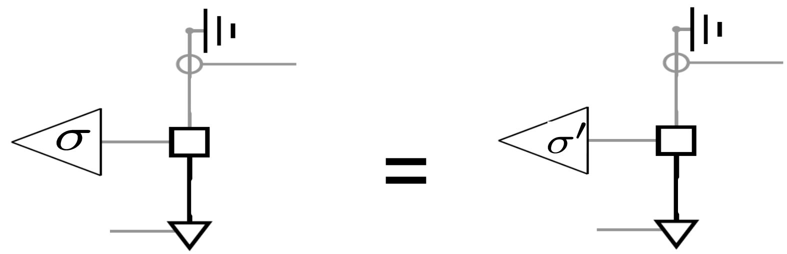
$$\sigma' = \frac{1}{2}[f_{\text{reset}-0}] + \frac{1}{2}[f_{\text{reset}-1}]$$

No causal influence



And yet,





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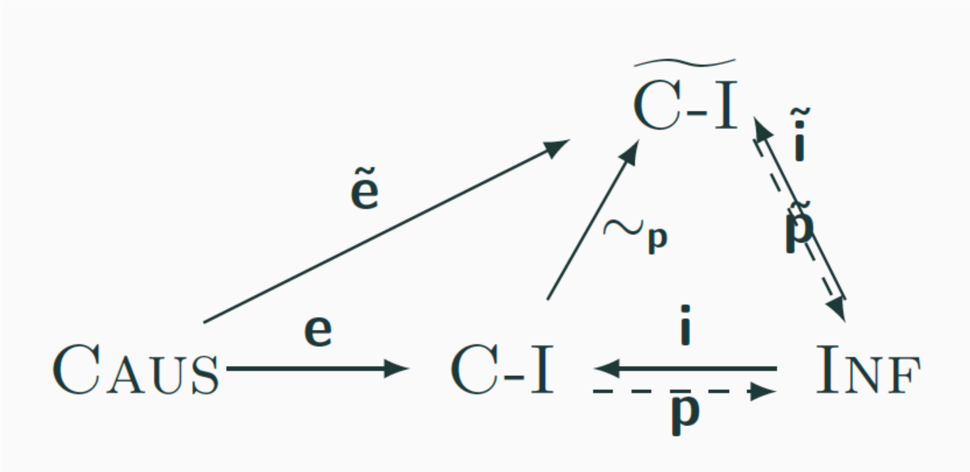
$$= \frac{1}{2}\begin{pmatrix} 1 & 1 \\ 0 & 0 \end{pmatrix} + \frac{1}{2}\begin{pmatrix} 0 & 0 \\ 1 & 1 \end{pmatrix}$$

Quotiented theories lose information about causal relations

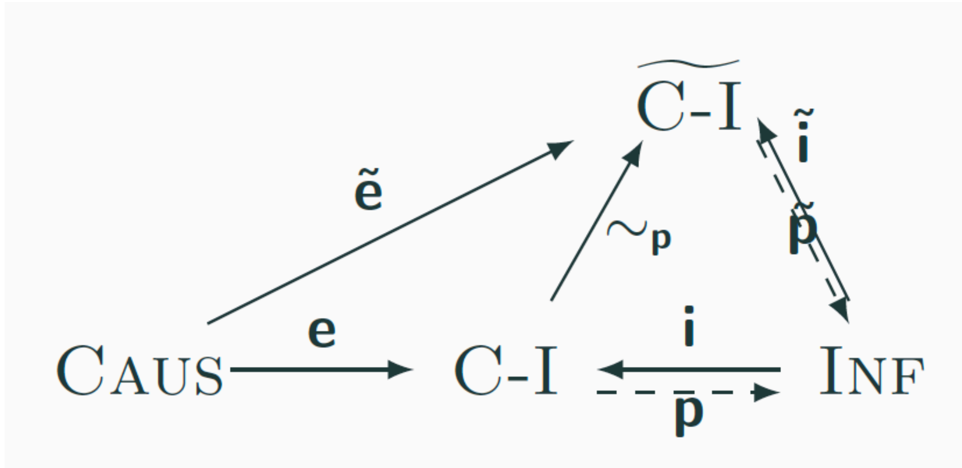
Quotienting

– This equivalence relation, $\sim_{\mathbf{p}}$, is preserved by composition, hence, we can quotient by it:

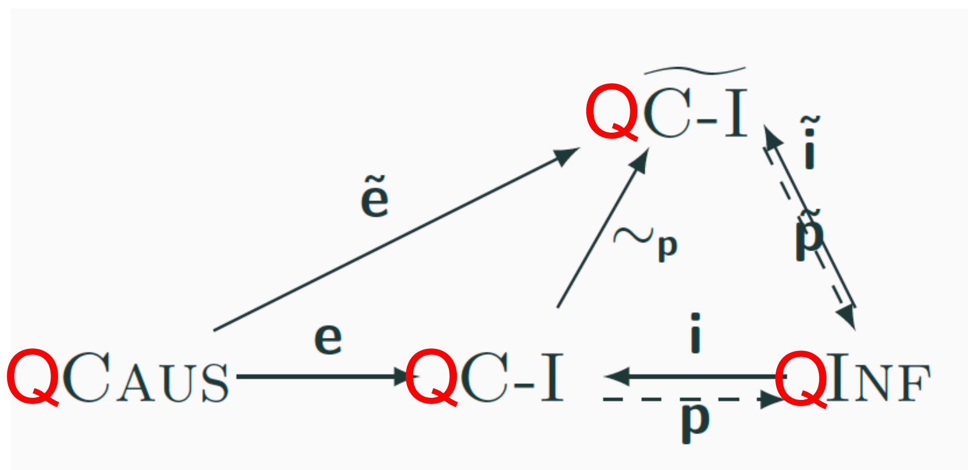


Because the quotiented theory scrambles causal and inferential notions, we must work with the unquotiented theory if we are to unscramble the omelette

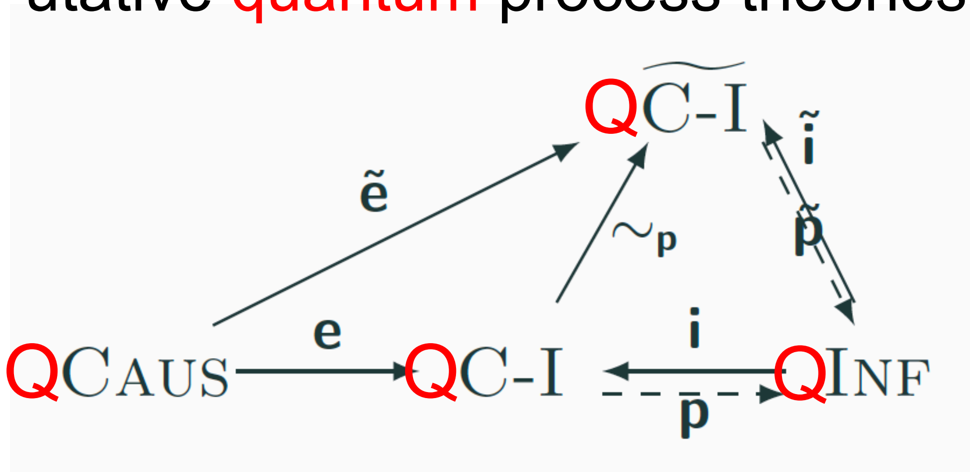
Classical process theories



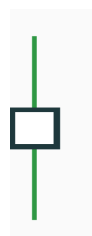
Putative quantum process theories



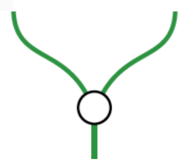
Putative quantum process theories



- functions → isometries

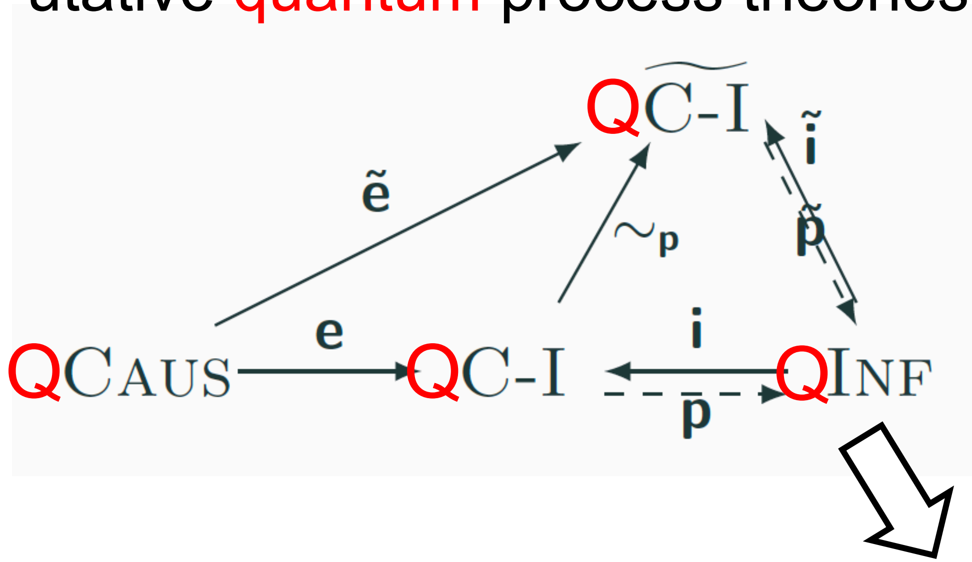


- Copy operation → partitioning (no physical broadcasting)



Allen, Barrett, Horsman, Lee, RWS, PRX 7, 031021 (2017)
Lorenz & Barrett, arXiv:2001.07774 (2020)

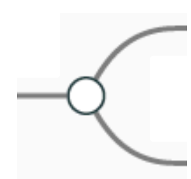
Putative quantum process theories



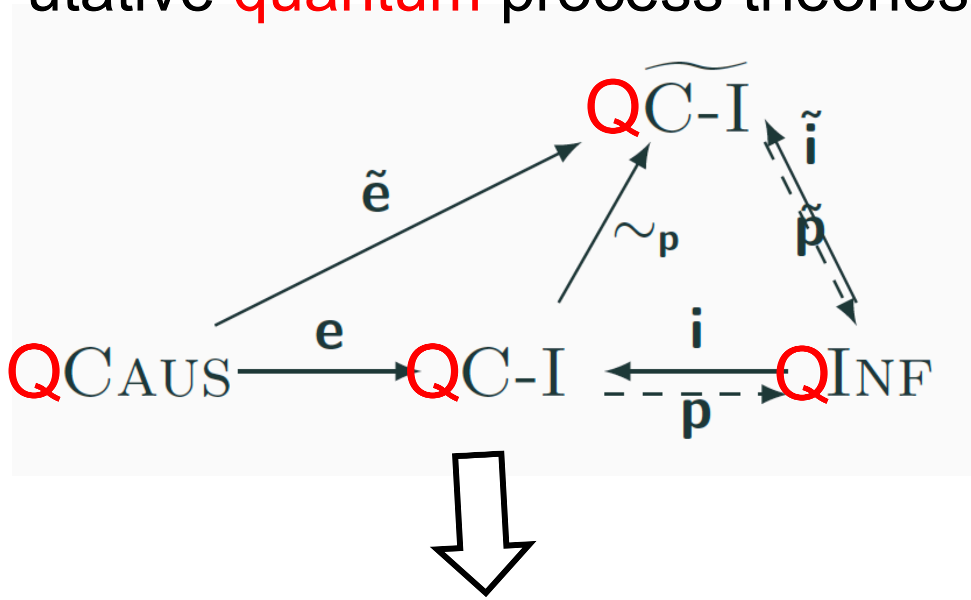
New type of quantum logic
New type of quantum Bayesian inference

- Conditioning on a variable → acquiring incomplete info about a system

- Logical broadcasting map



Putative quantum process theories



Interaction constrains the possibilities

Draft in preparation

Thanks for your attention!